Explain Boltzmann entropy relation.

In 1896 Boltzmann established a relation between entropy and Probability. The equilibrium state of the system is the state of maximum Probability that is the Probability of the system in equilibrium state is maximum. But from the thermodynamics, the equilibrium state of the system is the state of maximum entropy when a system not in equilibrium, changes take place within the system until the equilibrium state or the state of maximum entropy is reached In this way equilibrium state or the state of maximum entropy is reached. In this way equilibrium state both the entropy and thermodynamical Probability both have their maximum values. Boltzmann concluded that entropy is a function of Probability Ω.

$$S = f(\Omega) \text{ ------ (1)}$$

We consider two completely independent

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system A and B having entropies S_1 and S_2 respectively. since entropy is an additive quantity therefore the entropy S of two system together will be

$$S = S_1 + S_2 \quad \text{---(2)}$$

Let Ω_1 and Ω_2 be the probabilities of the system A and B respectively, then the probability Ω of finding both the system at their respective given condition will be

$$\Omega = \Omega_1 \times \Omega_2 \quad \text{---(3)}$$

Thus $S = f(\Omega) = f(\Omega_1) + f(\Omega_2)$ and

$$S_2 = f(\Omega_2)$$

Substituing these values in equation (2) g

$\Omega_2 f'(\Omega_1, \Omega_2) = f'(\Omega_1)$ and $\Omega_1 f'(\Omega_1, \Omega_2) = f'(\Omega_2)$

These equation give the Beta Paramet

Some standard (theorems)

$$\frac{f'(\Omega_1)}{f'(\Omega_2)} = \frac{\Omega_2}{\Omega_1}$$

or, $\Omega_1 f'(\Omega_1) = \Omega_2 f'(\Omega_2) = \cdots = K$

where K is any constant.

This gives $f'(\Omega_1) = \frac{K}{\Omega_1}$, $f'(\Omega_2) = \frac{K}{\Omega_2}$

Integrating it we get

$$f'(\Omega_1) = K \log \Omega_1 + C_1 \text{ and } f'(\Omega_2) = K \log \Omega_2 + C_2$$

In general, we can write $f(\Omega) = K \log \Omega + C$

or $S = K \log \Omega + C$ from eq (1) ---(5)

The constant of integration is chosen in accordance with the third law of thermodynamics.

for a thermodynamical system at absoute zero $\Omega = 1$ and $S = 0$

Therefore equation (5) give $C = 0$

$$\therefore \boxed{S = K \log \Omega}$$

This is the relation between entropy and probability.

constant K is known as Boltzmann Constant.